

Indian Statistical Institute, Bangalore

M. Math. Second Year

First Semester - Number Theory

Semestral Exam

Date : Nov 10, 2014

Note: Each question carries 20 marks - Full marks:100

Duration: 3 hours

1. Prove that any two positive integers x and n satisfy the congruence
$$\sum_{d|n} \mu(n/d)x^d \equiv 0 \pmod{n}$$
2. Using Gauss's law of Quadratic reciprocity and Dirichlet's theorem on primes in arithmetic progression (without proof) show that if an integer n is a square modulo every prime, then n is a perfect square.
3. Prove that every infinite continued fraction expansion converges to an irrational number.
4. Show that the sum of reciprocals of the prime numbers diverges.
5. Find a necessary and sufficient condition on a prime number p for the existence of integers x and y such that $p = x^2 - xy + y^2$.